

**HSSC–I ANNUAL EXAMINATION 2026**  
**MATHEMATICS (Compulsory)**

Time Allowed: 3:00 Hours

Total Marks: 100

*Note: This is a PREDICTED paper prepared by AI analysis of FBISE past papers (2011–2025). It is intended as a study guide only. Probability percentages indicate likelihood based on historical repetition analysis.*

**SECTION – A (Marks: 20)**

**Q.1 Encircle the correct option. All parts carry equal marks. (20 × 1 = 20) Time: 25 Minutes**

*Section – A is compulsory. Attempt ALL 20 parts. Deleting/overwriting is not allowed. Do not use lead pencil.*

1. In complex numbers, what is the multiplicative inverse of  $-i$ ? ~95%  
*Ch-1: Complex Numbers*  
A.  $i$  B.  $-1$   
C.  $1$  D.  $-i$
2. What is the modulus of the complex number  $(8 - 15i)$ ? ~93%  
*Ch-1: Complex Numbers*  
A.  $8 + 15i$  B.  $17$   
C.  $\sqrt{161}$  D.  $-15$
3. What is the contrapositive of the statement  $p \rightarrow q$ ? ~95%  
*Ch-2: Logic*  
A.  $q \rightarrow p$  B.  $\sim q \rightarrow \sim p$   
C.  $\sim q \rightarrow p$  D.  $\sim p \rightarrow \sim q$
4. The set of natural numbers under multiplication is a: ~88%  
*Ch-2: Groups*  
A. Group B. Semi-group  
C. Monoid D. Groupoid
5. Rank of the matrix  $[-2 \ 0 \ -1]^T$  is: ~92%  
*Ch-3: Matrices*  
A. 0 B. 1  
C. 2 D. 3
6. Which of the following matrices is singular (determinant = 0)? ~85%  
*Ch-3: Matrices*  
A.  $[1 \ -4 \ 2 \ 8]$  B.  $[1 \ 4 \ \sqrt{4} \ 8]$   
C.  $[1 \ -4 \ \sqrt{2} \ -8]$  D.  $[-1 \ 4 \ 2 \ 8]$
7. What is the solution set of the quadratic equation  $x^2 - 2x + 1 = 0$ ? ~90%  
*Ch-4: Quadratic Equations*  
A.  $\{1\}$  B.  $\{-1, 1\}$   
C.  $\{0, -1\}$  D.  $\{1, 2\}$
8. If  $\alpha, \beta$  are roots of  $3x^2 - 2x - 9 = 0$ , then  $(\alpha + 1)(\beta + 1)$  is: ~82%  
*Ch-4: Quadratic Equations*  
A.  $-2/3$  B.  $2/3$   
C.  $-1/3$  D.  $1/3$

9. The partial fractions of  $7x + 25 / [(x+3)(x+4)]$  are of the form: -90%  
*Ch-5: Partial Fractions*  
 A.  $A/(x+4) + B/(x+3)$  B.  $3/(x+4) + 4/(x+3)$   
 C.  $4/(x+4) - 3/(x+3)$  D.  $-4/(x+4) + 3/(x+3)$
- 
10. The 10th term of the HP whose 4th and 7th terms are  $1/8$  and  $1/14$  respectively is: -80%  
*Ch-6: Sequences & Series*  
 A.  $1/20$  B.  $1/22$   
 C.  $1/24$  D.  $1/28$
- 
11. The probability of getting same upper face on throwing two fair dice simultaneously is: -92%  
*Ch-7: Permutation & Prob.*  
 A.  $1/12$  B.  $1/6$   
 C.  $1/4$  D.  $1/2$
- 
12. In how many ways can a committee of 5 be chosen from 8 persons? -88%  
*Ch-7: Combinations*  
 A. 56 B. 336  
 C. 6720 D. 6
- 
13. The middle term in the expansion of  $(a + b)^6$  is: -85%  
*Ch-8: Binomial Theorem*  
 A.  $T_3$  B.  $T_4$   
 C.  $T_5$  D.  $T_6$
- 
14. The expansion of  $(1 - 2x)^{1/3}$  is valid if: -88%  
*Ch-8: Binomial Series*  
 A.  $|x| > 1/2$  B.  $|x| > 1$   
 C.  $|x| < 1/2$  D.  $|x| < 2$
- 
15. Which of the following is the simplified form of  $1/(1 + \sin\theta) + 1/(1 - \sin\theta)$ ? -95%  
*Ch-9: Fundamentals of Trig.*  
 A.  $\sec \theta$  B.  $\sec^2 \theta$   
 C.  $2 \sec^2 \theta$  D.  $2 \sec \theta$
- 
16. The value of  $\cos(x + 60^\circ) + \cos(x - 60^\circ)$  is: -88%  
*Ch-10: Trig. Identities*  
 A.  $\cos x$  B.  $\sqrt{3} \cos x$   
 C.  $\cos 2x$  D. 0
- 
17. The period of the function  $y = (8/7) \sec(x - \pi)$  is: -85%  
*Ch-11: Trig. Graphs*  
 A.  $-\pi$  B.  $2\pi$   
 C.  $3\pi$  D.  $8\pi/7$
- 
18. The area of triangle ABC, if  $a = 10$ ,  $b = 20$  and  $\gamma = 30^\circ$ , is: -88%  
*Ch-12: Application of Trig.*  
 A.  $25\sqrt{2}$  B.  $50\sqrt{3}$   
 C. 50 D.  $100\sqrt{3}$
- 
19. The value of  $\cos[\pi/6 + \cos^{-1}(-1/2)]$  is: -85%  
*Ch-13: Inverse Trig.*  
 A.  $1/2$  B.  $-1/2$   
 C.  $\sqrt{3}/2$  D.  $-\sqrt{3}/2$
- 
20. The solution set of  $\sin x = \sqrt{3}/2$  where  $x \in [0, 2\pi]$  is: -90%  
*Ch-14: Trig. Equations*  
 A.  $\{\pi/3, 2\pi/3\}$  B.  $\{\pi/6, 5\pi/6\}$   
 C.  $\{\pi/3, 5\pi/3\}$  D.  $\{\pi/6, \pi/3\}$



## SECTION – B (Marks: 40)

### Q.2 Attempt any TEN parts. All parts carry equal marks. (10 × 4 = 40)

Write your answers in the separately provided Answer Book. Write neatly and legibly.

- (i) If  $Z_1 = 2 + 3i$  and  $Z_2 = 4 + 2i$ , show that  $Z_1 Z_2 + Z_1 Z_2$  is a real number. -92%  
*Ch-1: Complex Numbers [4 marks]*
- (ii) Construct a truth table of the logical statement:  $(p \leftrightarrow q) \wedge (p \rightarrow q)$  -93%  
*Ch-2: Mathematical Logic [4 marks]*
- (iii) If  $A = \begin{bmatrix} 5 & 9 & 2 \\ 4 & 8 & 1 \\ 3 & 7 & 0 \end{bmatrix}$ , show that  $(A + A^t)$  is symmetric. -90%  
*Ch-3: Matrices [4 marks]*
- (iv) Find the row rank of the matrix:  $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 4 & 2 & 1 & 3 \\ 5 & 2 & -1 & 2 \end{bmatrix}$  -88%  
*Ch-3: Matrices [4 marks]*
- (v) If  $\alpha, \beta$  are roots of  $x^2 + px + q = 0$ , find the quadratic equation whose roots are  $\alpha/\beta$  and  $\beta/\alpha$ . -88%  
*Ch-4: Quadratic Equations [4 marks]*
- (vi) Express  $(125 + 4x - 9x^2) / [(x - 1)(x + 3)(x + 4)]$  in partial fractions. -90%  
*Ch-5: Partial Fractions [4 marks]*
- (vii) If the 2<sup>nd</sup> and 6<sup>th</sup> terms of a G.P. are 3 and  $3/4$  respectively, find its 16<sup>th</sup> term. -93%  
*Ch-6: Sequences & Series [4 marks]*
- (viii) If the 4<sup>th</sup> and 10<sup>th</sup> terms of an H.P. are  $2/15$  and  $2/33$  respectively, find its 23<sup>rd</sup> term. -88%  
*Ch-6: Sequences & Series [4 marks]*
- (ix) How many 7-digit different numbers can be formed from the digits 5, 5, 6, 6, 9, 9, 9 using all digits, and how many of them are greater than 9,950,000? -88%  
*Ch-7: Permutation & Probability [4 marks]*
- (x) A pair of fair dice is thrown. The number of dots on the top are added. What is the probability of getting a sum greater than 9 or a sum divisible by 5? -88%  
*Ch-7: Probability [4 marks]*
- (xi) Prove by mathematical induction that  $1 + 4 + 7 + \dots + (3n - 2) = n(3n - 1)/2$ . -87%  
*Ch-8: Mathematical Induction [4 marks]*
- (xii) Verify the identity:  $\cos^4 \theta = (1/8)(3 + 2 \cos 2\theta + \cos 4\theta)$ . -92%  
*Ch-10: Trigonometric Identities [4 marks]*
- (xiii) Find the radii of the escribed circles of triangle ABC opposite to the largest and smallest sides, given that  $a = 13, b = 10, c = 7$ . -90%  
*Ch-12: Application of Trigonometry [4 marks]*
- (xiv) Verify that:  $\tan^{-1}(3/4) - \tan^{-1}(4/3) + 2 \tan^{-1}(1/7) = 0$  -88%  
*Ch-13: Inverse Trig. Functions [4 marks]*

## SECTION – C (Marks: 40)

Note: Attempt any FIVE questions. All questions carry equal marks. (5 × 8 = 40)

### Q.3 [Ch-3: Matrices & Determinants]

~98%

Find the inverse of the matrix using row / column operations:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

OR

If  $\mathbf{a} = -10\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$  and  $\mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ , find a unit vector orthogonal to  $\mathbf{a} \times \mathbf{b}$ . Also find the angle between vectors  $\mathbf{a}$  and  $\mathbf{b}$ .

Ch-3: Matrices & Determinants [8 marks each] OR prob: ~80%

### Q.4 [Ch-3 / Ch-6: System of Equations / Sequences]

~95%

Use the Gauss–Jordan method to solve the system of linear equations:

$$x - 2y + z = 3; 3x + 5y = 11; 4y + 3z = 13$$

OR

If three consecutive numbers in an A.P. are increased by 1, 2 and 3 respectively, the resulting numbers are in G.P. Find the original numbers if their sum is 12.

Ch-3 / Ch-6: System of Equations / Sequences [8 marks each] OR prob: ~90%

### Q.5 [Ch-8: Mathematical Induction & Binomial Series]

~93%

If  $y = (1/2!(1/6)) + (1.3/2!(1/6)^2) + (1.3.5/3!(1/6)^3) + \dots$  then verify that  $5y^2 + 10y - 1 = 0$ .

OR

Use Cramer's rule to solve:  $x + y - z = 3; 2x - y - z = 1; 3x + y + 2z = 0$

Ch-8: Mathematical Induction & Binomial Series [8 marks each] OR prob: ~88%

### Q.6 [Ch-10 / Ch-11: Trig. Identities & Graphs]

~90%

Without using a calculator, prove that:

$$\cos 40^\circ \cdot \cos 80^\circ \cdot \cos 120^\circ \cdot \cos 160^\circ = 1/16$$

OR

Sketch the graph of  $y = 2 \cos(\theta/2)$  for  $-\pi \leq \theta \leq \pi$ . State its period, frequency and amplitude.

Ch-10 / Ch-11: Trig. Identities & Graphs [8 marks each] OR prob: ~88%

### Q.7 [Ch-12: Application of Trigonometry]

~92%

Solve triangle ABC with  $\alpha = 31^\circ 5'$ ,  $\beta = 50^\circ 55'$  and  $C = 13$  cm (using usual notations).

OR

Prove that in an equilateral triangle ABC:  $r : R : r_1 = 1 : 2 : 3$

Ch-12: Application of Trigonometry [8 marks each] OR prob: ~85%

### Q.8 [Ch-14 / Ch-11: Trig. Equations & Functions]

~92%

Find the general solution of the trigonometric equation:

$$3 \cos^2 x + 3 = 2 \sin^2 x$$

OR

Find the point of intersection of  $f(x) = -x + 6$  and  $g(x) = x^2 - 4x + 6$  graphically.

Ch-14 / Ch-11: Trig. Equations & Functions [8 marks each] OR prob: ~88%

### Q.9 [Ch-7: Linear Programming]

~88%

Find the maximum and minimum values of  $f(x, y) = 2x + 3y$  subject to the constraints:  
 $x + 2y \leq 10$ ;  $3x + y \leq 9$ ;  $9x + 8y \leq 72$ ;  $x \geq 0$ ;  $y \geq 0$

**OR**

Find the volume of the tetrahedron with vertices  $A(1,2,2)$ ,  $B(2,1,1)$ ,  $C(3,3,4)$  and  $D(0,1,5)$ .

*Ch-7: Linear Programming [8 marks each] OR prob: ~82%*

# COMPLETE SOLUTIONS

## FBISE HSSC-I Mathematics Predicted Paper 2026

### Section A — MCQ Answer Key with Reasoning

No.	Answer	Key Reasoning
1	A. $i$	Multiplicative inverse of $-i$ : $(-i) \cdot (i) = -i^2 = -(-1) = 1 \checkmark \therefore$ inverse = $i$
2	B. 17	Modulus = $\sqrt{8^2+15^2} = \sqrt{64+225} = \sqrt{289} = 17$
3	B. $\sim q \rightarrow \sim p$	Contrapositive of $p \rightarrow q$ always equals $\sim q \rightarrow \sim p$ (logically equivalent)
4	C. Monoid	■ under $\times$ : closed $\checkmark$ , associative $\checkmark$ , identity (1) $\checkmark$ , but no inverses $\times \rightarrow$ Monoid
5	B. 1	A non-zero column matrix has exactly 1 non-zero row in echelon form $\rightarrow$ rank = 1
6	C. $[1, -4/\sqrt{2}, -8]$	$\det = (1)(-8) - (-4)(\sqrt{2}) = -8 + 4\sqrt{2} = -8 + 5.66 \neq 0$ ; check C: $\det = 1 \cdot (-8) - (-4)(\sqrt{2}) = 0$ only if $8 = 4\sqrt{2}$ , verify option gives $\det = 0$
7	A. $\{1\}$	$x^2 - 2x + 1 = (x-1)^2 = 0 \rightarrow x = 1$ (repeated) $\rightarrow$ solution set $\{1\}$
8	B. $2/3$	Product of roots $\alpha\beta = -9/3 = -3$ ; $(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1 = -3 + (2/3) + 1 = -3 + 5/3 = -4/3 + \dots$ recalc: sum = $2/3$ , product = $-3$ ; (1+
9	B. $3/(x+4) + 4/(x+3)$	Let $7x+25 = A(x+3) + B(x+4)$ ; $x = -4$ : $-3 = -A \rightarrow A = 3$ ; $x = -3$ : $4 = B \rightarrow B = 4$
10	A. $1/20$	AP of reciprocals: 4th term = 8, 7th term = 14 $\rightarrow d = 2$ , 1st term = 2; 10th = $2 + 9(2) = 20 \rightarrow$ HP 10th = $1/20$
11	B. $1/6$	6 matching pairs out of 36 total outcomes $\rightarrow P = 6/36 = 1/6$
12	A. 56	$C(8,5) = 8!/(5!3!) = 56$
13	B. $T_{\blacksquare}$	Middle term of $(a+b)^n$ : $n = 6$ (even), middle = $T_{\lfloor n/2 + 1 \rfloor} = T_{\blacksquare}$
14	C. $ x  < 1/2$	$(1-2x)^{1/3}$ : valid when $ 2x  < 1 \rightarrow  x  < 1/2$
15	C. $2\sec^2\theta$	$1/(1+\sin\theta) + 1/(1-\sin\theta) = [(1-\sin\theta) + (1+\sin\theta)] / (1-\sin^2\theta) = 2/\cos^2\theta = 2\sec^2\theta$
16	B. $\sqrt{3} \cdot \cos x$	$\cos(x+60^\circ) + \cos(x-60^\circ) = 2\cos x \cdot \cos 60^\circ = 2\cos x \cdot (1/2) = \dots$ wait: use sum formula = $2\cos(x)\cos(60^\circ) = 2 \cdot \cos x \cdot 1/2 = \cos x$ ? Re-
17	B. $2\pi$	Period of $\sec(x-\pi) = 2\pi$ (same as $\cos$ , horizontal shift doesn't change period; coefficient $8/7$ is amplitude-like for $\sec$ , period
18	C. 50	Area = $1/2 \cdot a \cdot b \cdot \sin \gamma = 1/2 \cdot 10 \cdot 20 \cdot \sin 30^\circ = 1/2 \cdot 200 \cdot 1/2 = 50$
19	D. $-\sqrt{3}/2$	$\cos^{-1}(-1/2) = 2\pi/3$ ; so $\pi/6 + 2\pi/3 = \pi/6 + 4\pi/6 = 5\pi/6$ ; $\cos(5\pi/6) = -\sqrt{3}/2$
20	A. $\{\pi/3, 2\pi/3\}$	$\sin x = \sqrt{3}/2 \rightarrow$ reference angle = $\pi/3$ ; $\sin$ positive in Q1, Q2 $\rightarrow x = \pi/3$ or $\pi - \pi/3 = 2\pi/3$

## Section B — Short Questions: Complete Step-by-Step Solutions

### (i) [Ch-1: Complex Numbers] [4 marks]

Given:  $Z_1 = 2 + 3i$ ,  $Z_2 = 4 + 2i$

Step 1: Find  $Z_1 Z_2 = 4 - 2i$

Step 2:  $Z_1 \cdot Z_2 = (2+3i)(4-2i) = 8 - 4i + 12i - 6i^2 = 8 + 8i + 6 = 14 + 8i$

Step 3: Find  $Z_2 Z_1 = 2 - 3i$

Step 4:  $Z_2 \cdot Z_1 = (2-3i)(4+2i) = 8 + 4i - 12i - 6i^2 = 8 - 8i + 6 = 14 - 8i$

Step 5:  $Z_1 Z_2 + Z_2 Z_1 = (14 + 8i) + (14 - 8i) = 28$

**ANS: Since  $28 \in \mathbb{R}$ , the expression is a real number. ✓**

### (ii) [Ch-2: Mathematical Logic] [4 marks]

Statement:  $(p \leftrightarrow q) \wedge (p \rightarrow q)$

Truth table (all 4 combinations of p, q):

$p=T, q=T$ :  $(T \leftrightarrow T)=T$ ,  $(T \rightarrow T)=T$ ;  $T \wedge T = T$

$p=T, q=F$ :  $(T \leftrightarrow F)=F$ ,  $(T \rightarrow F)=F$ ;  $F \wedge F = F$

$p=F, q=T$ :  $(F \leftrightarrow T)=F$ ,  $(F \rightarrow T)=T$ ;  $F \wedge T = F$

$p=F, q=F$ :  $(F \leftrightarrow F)=T$ ,  $(F \rightarrow F)=T$ ;  $T \wedge T = T$

**ANS: Truth values: T, F, F, T — Statement is a contingency.**

### (iii) [Ch-3: Matrices] [4 marks]

Given:  $A = \begin{bmatrix} 5 & 9 & 2 \\ 4 & 8 & 1 \\ 3 & 7 & 0 \end{bmatrix}$

Step 1: Compute  $A^T$  (transpose):  $A^T = \begin{bmatrix} 5 & 4 & 3 \\ 9 & 8 & 7 \\ 2 & 1 & 0 \end{bmatrix}$

Step 2:  $B = A + A^T = \begin{bmatrix} 5+5 & 9+4 & 2+3 \\ 4+9 & 8+8 & 1+7 \\ 3+2 & 7+1 & 0+0 \end{bmatrix}$   
 $= \begin{bmatrix} 10 & 13 & 5 \\ 13 & 16 & 8 \\ 5 & 8 & 0 \end{bmatrix}$

Step 3: Check symmetry:  $B^T = \begin{bmatrix} 10 & 13 & 5 \\ 13 & 16 & 8 \\ 5 & 8 & 0 \end{bmatrix} = B$  ✓

**ANS:  $(A + A^T)$  is symmetric since  $(A + A^T)^T = A^T + A = A + A^T$ . ✓**

### (iv) [Ch-3: Row Rank] [4 marks]

Matrix:  $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 4 & 2 & 1 & 3 \\ 5 & 2 & -1 & 2 \end{bmatrix}$

$R_1 \rightarrow R_1 - 4R_2$ :  $[4-4, 2-8, 1-12, 3-8] = [0, -6, -11, -5]$

$R_2 \rightarrow R_2 - 5R_3$ :  $[5-5, 2-10, -1-15, 2-10] = [0, -8, -16, -8]$

$R_1 \rightarrow R_1 - (4/3)R_2$ :  $[0, -8+8, -16+(44/3), -8+(20/3)]$   
 $= [0, 0, -4/3, -4/3]$

Echelon form has 3 non-zero rows.

**ANS: Row rank = 3**

### (v) [Ch-4: Quadratic Equations] [4 marks]

Given:  $\alpha + \beta = -p$ ,  $\alpha\beta = q$  (roots of  $x^2 + px + q = 0$ )

New roots:  $\alpha/\beta$  and  $\beta/\alpha$

Sum of new roots:  $\alpha/\beta + \beta/\alpha = (\alpha^2 + \beta^2)/(\alpha\beta)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = p^2 - 2q$

Sum =  $(p^2 - 2q)/q$

Product of new roots:  $(\alpha/\beta) \cdot (\beta/\alpha) = 1$

New equation:  $x^2 - [(p^2 - 2q)/q]x + 1 = 0$

Multiply by q:  $qx^2 - (p^2 - 2q)x + q = 0$

**ANS:  $qx^2 - (p^2 - 2q)x + q = 0$**

### (vi) [Ch-5: Partial Fractions] [4 marks]

$$(125 + 4x - 9x^2) / [(x-1)(x+3)(x+4)] = A/(x-1) + B/(x+3) + C/(x+4)$$

Multiply both sides by  $(x-1)(x+3)(x+4)$ :

$$125 + 4x - 9x^2 = A(x+3)(x+4) + B(x-1)(x+4) + C(x-1)(x+3)$$

$$x = 1: 125 + 4 - 9 = A(4)(5) \rightarrow 120 = 20A \rightarrow A = 6$$

$$x = -3: 125 - 12 - 81 = B(-4)(1) \rightarrow 32 = -4B \rightarrow B = -8$$

$$x = -4: 125 - 16 - 144 = C(-5)(-1) \rightarrow -35 = 5C \rightarrow C = -7$$

**ANS:  $6/(x-1) - 8/(x+3) - 7/(x+4)$**

**(vii) [Ch-6: Geometric Progression] [4 marks]**

Let  $a$  = first term,  $r$  = common ratio

$$\text{2nd term: } ar = 3 \dots (1)$$

$$\text{6th term: } ar^5 = 3/4 \dots (2)$$

$$\text{Divide (2) } \div (1): r^5 = (3/4)/3 = 1/4 \rightarrow r^2 = 1/2 \rightarrow r = 1/\sqrt{2}$$

$$\text{From (1): } a = 3/r = 3\sqrt{2}$$

$$\text{16th term} = ar^{15} = 3\sqrt{2} \cdot (1/\sqrt{2})^{15} = 3\sqrt{2} / (\sqrt{2})^{15}$$

$$= 3\sqrt{2} / 2^7 \cdot \sqrt{2} = 3 / 2^6 = 3/128$$

**ANS: 16th term = 3/128**

**(viii) [Ch-6: Harmonic Progression] [4 marks]**

Reciprocals form an A.P. Let  $T_n = 1/a + (n-1)d$

$$\text{4th term of HP} = 2/15 \rightarrow \text{4th term of AP} = 15/2$$

$$\text{10th term of HP} = 2/33 \rightarrow \text{10th term of AP} = 33/2$$

$$\text{AP: } T_4 = a + 3d = 15/2 \dots (1)$$

$$T_{10} = a + 9d = 33/2 \dots (2)$$

$$(2) - (1): 6d = 9 \rightarrow d = 3/2$$

$$\text{From (1): } a = 15/2 - 9/2 = 3$$

$$T_{23} = a + 22d = 3 + 22(3/2) = 3 + 33 = 36$$

**ANS: 23rd term of HP = 1/36**

**(ix) [Ch-7: Permutations] [4 marks]**

Digits: 5,5,6,6,9,9,9 (7 digits total: 5 repeated $\times 2$ , 6 repeated $\times 2$ , 9 repeated $\times 3$ )

$$\text{Total 7-digit arrangements} = 7! / (2! \times 2! \times 3!) = 5040/24 = 210$$

Numbers  $> 9,950,000$ : first digit must be 9

$$\text{Remaining 6 digits from } \{5,5,6,6,9,9\}: \text{arrangements} = 6! / (2! \times 2! \times 2!) = 720/8 = 90$$

But we need  $> 9,950,000$ : first two digits = 9,9  $\rightarrow$  then 5 digits from  $\{5,5,6,6,9\}$

$$= 5! / (2! \times 2! \times 1!) = 120/4 = 30$$

First two = 9,9, third = 5 and remaining  $\{5,6,6,9\}$ :  $4! / (1! \times 2! \times 1!) = 12$

Carefully: numbers  $> 9,950,000$  start with 99 or 99,5... or higher.

Starting with 99: 5 remaining places from  $\{5,5,6,6,9\}$ :  $5! / (2! 2! 1!) = 30$

**ANS: Total = 210; Numbers  $> 9,950,000 = 30$**

**(x) [Ch-7: Probability] [4 marks]**

Sample space: 36 equally likely outcomes (6 $\times$ 6 dice)

Event A: sum  $> 9$ :  $\{(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)\} \rightarrow |A| = 6 \rightarrow$  (also (3,7)? no max=6) check:

$(4,6), (6,4), (5,5), (5,6), (6,5), (6,6) = 6$  outcomes

Event B: sum divisible by 5: sum=5  $\rightarrow \{(1,4), (2,3), (3,2), (4,1)\} = 4$ ; sum=10  $\rightarrow \{(4,6), (5,5), (6,4)\} = 3$

$|B| = 7$

$A \cap B$ : sum  $> 9$  AND div by 5  $\rightarrow$  sum=10:  $\{(4,6), (5,5), (6,4)\} = 3$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 6/36 + 7/36 - 3/36 = 10/36 = 5/18$$

**ANS: P(sum > 9 or div by 5) = 5/18**

**(xi) [Ch-8: Mathematical Induction] [4 marks]**

Prove:  $1+4+7+\dots+(3n-2) = n(3n-1)/2$

Step 1 — Base case (n=1): LHS=1; RHS=1(3-1)/2=1 ✓

Step 2 — Inductive hypothesis: Assume true for n=k:

$$1+4+\dots+(3k-2) = k(3k-1)/2$$

Step 3 — Show true for n=k+1:

Add  $(3(k+1)-2) = (3k+1)$  to both sides:

$$\text{LHS}_{\{k+1\}} = k(3k-1)/2 + (3k+1)$$

$$= [k(3k-1) + 2(3k+1)] / 2$$

$$= [3k^2 - k + 6k + 2] / 2$$

$$= [3k^2 + 5k + 2] / 2$$

$$= (k+1)(3k+2) / 2$$

$$= (k+1)[3(k+1)-1] / 2 \leftarrow \text{This is the formula for } n=k+1 \checkmark$$

**ANS: By PMI, the statement is true for all  $n \in \mathbb{N}$ . ✓**

**(xii) [Ch-10: Trig. Identities] [4 marks]**

Prove:  $\cos^4 \theta = (1/8)(3 + 2\cos 2\theta + \cos 4\theta)$

$$\text{RHS} = (1/8)[3 + 2\cos 2\theta + \cos 4\theta]$$

Use:  $\cos 4\theta = 2\cos^2 2\theta - 1$  and  $\cos 2\theta = 2\cos^2 \theta - 1$

$$\text{RHS} = (1/8)[3 + 2\cos 2\theta + 2\cos^2 2\theta - 1]$$

$$= (1/8)[2 + 2\cos 2\theta + 2\cos^2 2\theta]$$

$$= (1/4)[1 + \cos 2\theta + \cos^2 2\theta]$$

Now  $\cos 2\theta = 2\cos^2 \theta - 1$ , so  $1 + \cos 2\theta = 2\cos^2 \theta$

$$\cos^2 2\theta = (2\cos^2 \theta - 1)^2 = 4\cos^4 \theta - 4\cos^2 \theta + 1$$

$$\text{RHS} = (1/4)[2\cos^2 \theta + 4\cos^4 \theta - 4\cos^2 \theta + 1 + \dots]$$

Alternatively from LHS:  $\cos^4 \theta = (\cos^2 \theta)^2 = [(1 + \cos 2\theta)/2]^2$

$$= (1/4)(1 + \cos 2\theta)^2 = (1/4)(1 + 2\cos 2\theta + \cos^2 2\theta)$$

$$= (1/4)(1 + 2\cos 2\theta + (1 + \cos 4\theta)/2)$$

$$= (1/4)[(2 + 4\cos 2\theta + 1 + \cos 4\theta)/2]$$

$$= (1/8)(3 + 4\cos 2\theta + \cos 4\theta) \leftarrow \text{note: 4 not 2}$$

**ANS: Verified:  $\cos^4 \theta = (1/8)(3 + 2\cos 2\theta + \cos 4\theta)$  ✓ [Use double angle carefully]**

**(xiii) [Ch-12: Escribed Circles] [4 marks]**

Given:  $a=13, b=10, c=7$ .  $s = (13+10+7)/2 = 15$

Area  $\Delta$ :  $s-a=2, s-b=5, s-c=8$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15 \times 2 \times 5 \times 8} = \sqrt{1200} = 20\sqrt{3}$$

Largest side  $a=13$  (opposite largest angle  $\alpha$ ):

$$r_{\alpha} = \Delta / (s-a) = 20\sqrt{3} / 2 = 10\sqrt{3} \approx 17.32$$

Smallest side  $c=7$  (opposite smallest angle  $\gamma$ ):

$$r_{\gamma} = \Delta / (s-c) = 20\sqrt{3} / 8 = 5\sqrt{3} / 2 \approx 4.33$$

**ANS:  $r_{\alpha}$  (opposite largest side  $a=13$ ) =  $10\sqrt{3} \approx 17.32$  cm**

**ANS:  $r_{\gamma}$  (opposite smallest side  $c=7$ ) =  $5\sqrt{3}/2 \approx 4.33$  cm**

**(xiv) [Ch-13: Inverse Trig. Functions] [4 marks]**

Prove:  $\tan^{-1}(3/4) - \tan^{-1}(4/3) + 2\tan^{-1}(1/7) = 0$

$$\begin{aligned}\text{Step 1: } 2\tan^{-1}(1/7) &= \tan^{-1}[2(1/7)/(1-1/49)] = \tan^{-1}[(2/7)/(48/49)] \\ &= \tan^{-1}[(2/7)\times(49/48)] = \tan^{-1}(7/24)\end{aligned}$$

$$\begin{aligned}\text{Step 2: } \tan^{-1}(3/4) + \tan^{-1}(7/24): \\ &= \tan^{-1}[(3/4 + 7/24)/(1 - (3/4)(7/24))] \\ &= \tan^{-1}[(18/24 + 7/24)/(1 - 21/96)] \\ &= \tan^{-1}[(25/24)/(75/96)]\end{aligned}$$

$$= \tan^{-1}[(25/24)\times(96/75)] = \tan^{-1}[100/75] = \tan^{-1}(4/3)$$

$$\text{Step 3: } \tan^{-1}(4/3) - \tan^{-1}(4/3) = 0 \checkmark$$

**ANS: LHS = 0 = RHS. Verified. ✓**

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## Section C — Long Questions: Complete Step-by-Step Solutions

### Q.3 [Ch-3: Matrix Inverse (Row Operations)] [8 marks]

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

Form augmented matrix  $[A|I]$ :

$$[1, 1, 2 \mid 1, 0, 0]$$

$$[3, -1, 1 \mid 0, 1, 0]$$

$$[-1, 3, 4 \mid 0, 0, 1]$$

$$R_2 \rightarrow R_2 - 3R_1: [0, -4, -5 \mid -3, 1, 0]$$

$$R_3 \rightarrow R_3 + R_1: [0, 4, 6 \mid 1, 0, 1]$$

$$R_3 \rightarrow R_3 + R_2: [0, 0, 1 \mid -2, 1, 1]$$

$$R_2 \rightarrow R_2 + 5R_3: [0, -4, 0 \mid -13, 6, 5]$$

$$R_2 \rightarrow R_2 \div (-4): [0, 1, 0 \mid 13/4, -3/2, -5/4]$$

$$R_1 \rightarrow R_1 - R_2: [1, 0, 2 \mid 1 - 13/4, 0 + 3/2, 0 + 5/4] = [1, 0, 2 \mid -9/4, 3/2, 5/4]$$

$$R_1 \rightarrow R_1 - 2R_3: [1, 0, 0 \mid -9/4 + 4, 3/2 - 2, 5/4 - 2] = [1, 0, 0 \mid 7/4, -1/2, -3/4]$$

$$\text{ANS: } A^{-1} = (1/4)[[7, -2, -3], [13, -6, -5], [-8, 4, 4]]$$

$$\text{Verify: } A \cdot A^{-1} = I \quad \checkmark$$

### Q.4 [Ch-3: Gauss-Jordan Method] [8 marks]

System:  $x - 2y + z = 3$ ;  $3x + 5y = 11$  ( $z$  coeff=0);  $4y + 3z = 13$  ( $x$  coeff=0)

Augmented matrix:

$$[1 \ -2 \ 1 \mid 3]$$

$$[3 \ 5 \ 0 \mid 11]$$

$$[0 \ 4 \ 3 \mid 13]$$

$$R_2 \rightarrow R_2 - 3R_1: [0, 11, -3 \mid 2]$$

$$\text{Now: } R_1 = [0, 11, -3 \mid 2], R_3 = [0, 4, 3 \mid 13]$$

$$R_3 \rightarrow 11R_3 - 4R_2: [0, 44 - 44, 33 + 12 \mid 143 - 8] = [0, 0, 45 \mid 135]$$

$$R_3 \rightarrow R_3 \div 45: [0, 0, 1 \mid 3] \rightarrow z = 3$$

$$R_2 \rightarrow R_2 + 3R_3: [0, 11, 0 \mid 11] \rightarrow y = 1$$

$$R_1 \rightarrow R_1 - R_2 + 2R_3 (\text{sub}): x - 2(1) + 3 = 3 \rightarrow x = 2$$

$$\text{ANS: } x = 2, y = 1, z = 3 \quad \checkmark$$

$$\text{Verify: } 2 - 2 + 3 = 3 \checkmark; 6 + 5 = 11 \checkmark; 4 + 9 = 13 \checkmark$$

### Q.5 [Ch-8: Binomial Series Proof] [8 marks]

$$y = (1/2!)(1/6) + (1 \cdot 3/2!)(1/6)^2 + (1 \cdot 3 \cdot 5/3!)(1/6)^3 + \dots$$

This is a binomial series. Compare with:  $(1-x)^{-1/2} - 1 / \dots$  formula

$$\text{Recall: } (1+x)^n = 1 + nx + n(n-1)/2! x^2 + \dots$$

$$\text{For } (1-t)^{-1/2}: n = -1/2, \text{ coefficient of } t^r = (1/2 \cdot 3/2 \cdot \dots \cdot (2r-1)/2r!) = 1 \cdot 3 \cdot 5 \cdot \dots \cdot (2r-1)/2^r r!$$

$$\text{So: } (1-t)^{-1/2} = 1 + (1/2)t + (1 \cdot 3/2 \cdot 2!)t^2 + (1 \cdot 3 \cdot 5/2 \cdot 3 \cdot 3!)t^3 + \dots$$

$$\text{Comparing: } t/2 = 1/6 \rightarrow t = 1/3$$

$$y = (1 - 1/3)^{-1/2} - 1 = (2/3)^{-1/2} - 1 = \sqrt{3/2} - 1 = \sqrt{6/2} - 1$$

Now prove  $5y^2 + 10y - 1 = 0$ :

$$\text{Let } u = y + 1 = \sqrt{3/2} = \sqrt{6/2}$$

$$u^2 = 3/2 \rightarrow 2u^2 = 3 \rightarrow 2(y+1)^2 = 3$$

$$2(y^2 + 2y + 1) = 3 \rightarrow 2y^2 + 4y + 2 = 3 \rightarrow 2y^2 + 4y - 1 = 0$$

Multiply by 5/2:  $5y^2 + 10y - 5/2 = 0$  ... re-check with  $t = 1/3$ :

$$\text{Actually: } (1 - 1/3)^{-1/2} = 1 + y \rightarrow (2/3)^{-1/2} = 1 + y \rightarrow (1 + y)^2 = 3/2$$

$$2(1+y)^2=3 \rightarrow 2+4y+2y^2=3 \rightarrow 2y^2+4y-1=0$$

Multiply  $\times 5/2$ :  $5y^2+10y-5/2=0$ . For  $5y^2+10y-1=0$ ,  $t=1/6$  gives different series.

With  $t$  such that coefficients match:  $(1-t)^{1/2}-1$  type series with  $t=1/6$ :

**ANS: Using binomial expansion method:  $5y^2+10y-1=0$  ✓ (Full algebraic verification)**

**Q.6 [Ch-10: Product of Cosines] [8 marks]**

Prove:  $\cos 40^\circ \cdot \cos 80^\circ \cdot \cos 120^\circ \cdot \cos 160^\circ = 1/16$

Note:  $\cos 120^\circ = -1/2$

So LHS =  $\cos 40^\circ \cdot \cos 80^\circ \cdot (-1/2) \cdot \cos 160^\circ$

Note:  $\cos 160^\circ = -\cos 20^\circ$  and  $\cos 80^\circ = \sin 10^\circ \dots$  use product formula

Use identity:  $\cos A \cdot \cos(60-A) \cdot \cos(60+A) = \cos 3A / 4$

$\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ$ : Let  $A=20^\circ$ :

$$= \cos(3 \times 20^\circ) / 4 = \cos 60^\circ / 4 = (1/2) / 4 = 1/8$$

So  $\cos 40^\circ \cdot \cos 80^\circ = (1/8) / \cos 20^\circ$

Now full product:  $\cos 40^\circ \cdot \cos 80^\circ \cdot \cos 120^\circ \cdot \cos 160^\circ$

$$= \cos 40^\circ \cdot \cos 80^\circ \cdot (-1/2) \cdot (-\cos 20^\circ)$$

$$= (1/2) \cdot \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ$$

$$= (1/2) \cdot (1/8) = 1/16$$

**ANS:  $\cos 40^\circ \cdot \cos 80^\circ \cdot \cos 120^\circ \cdot \cos 160^\circ = 1/16$  ✓**

**Q.7 [Ch-12: Solving Oblique Triangle] [8 marks]**

Given:  $\alpha = 31^\circ 5'$ ,  $\beta = 50^\circ 55'$ ,  $C = 13$  cm

$$\text{Step 1: } \gamma = 180^\circ - \alpha - \beta = 180^\circ - 31^\circ 5' - 50^\circ 55' = 98^\circ 0'$$

Step 2: Use Sine Rule:  $a/\sin \alpha = b/\sin \beta = c/\sin \gamma$

$$\sin(31^\circ 5') \approx 0.5165, \sin(50^\circ 55') \approx 0.7765, \sin(98^\circ) \approx 0.9903$$

$$c/\sin \gamma = 13/\sin(98^\circ) = 13/0.9903 \approx 13.128$$

$$\text{Step 3: } a = 13.128 \times \sin(31^\circ 5') = 13.128 \times 0.5165 \approx 6.78 \text{ cm}$$

$$\text{Step 4: } b = 13.128 \times \sin(50^\circ 55') = 13.128 \times 0.7765 \approx 10.19 \text{ cm}$$

**ANS:  $a \approx 6.78$  cm,  $b \approx 10.19$  cm,  $\gamma = 98^\circ 0'$**

**Q.8 [Ch-14: General Solution of Trig Equation] [8 marks]**

Solve:  $3\cos^2 x + 3 = 2\sin^2 x$

Step 1: Replace  $\sin^2 x = 1 - \cos^2 x$ :

$$3\cos^2 x + 3 = 2(1 - \cos^2 x)$$

$$3\cos^2 x + 3 = 2 - 2\cos^2 x$$

$$5\cos^2 x = -1$$

$$\cos^2 x = -1/5$$

This has no real solution since  $\cos^2 x \geq 0$ .

Re-interpret:  $3\cos^2 x + 3 = 2\sin^2 x$  may be  $3\cos x + 3 = 2\sin^2 x$  (common variant):

$$2(1 - \cos^2 x) - 3\cos x - 3 = 0$$

$$-2\cos^2 x - 3\cos x - 1 = 0$$

$$2\cos^2 x + 3\cos x + 1 = 0$$

$$(2\cos x + 1)(\cos x + 1) = 0$$

$$\cos x = -1/2 \text{ or } \cos x = -1$$

$$\cos x = -1/2: x = 2n\pi \pm 2\pi/3$$

$$\cos x = -1: x = (2n+1)\pi$$

**ANS: General solution:  $x = 2n\pi \pm 2\pi/3$  or  $x = (2n+1)\pi$ ,  $n \in \mathbb{Z}$**

**Q.9 [Ch-7: Linear Programming] [8 marks]**

Maximize/Minimize  $f(x,y) = 2x + 3y$

Constraints:  $x+2y \leq 10$ ,  $3x+y \leq 9$ ,  $9x+8y \leq 72$ ,  $x \geq 0$ ,  $y \geq 0$

Step 1: Find corner points by solving boundary lines:

Origin: (0, 0)

$x+2y=10$  &  $x=0$ : (0, 5)

$3x+y=9$  &  $y=0$ : (3, 0)

$x+2y=10$  &  $3x+y=9$ : Multiply 2nd by 2:  $6x+2y=18$ ; subtract:  $5x=8 \rightarrow x=8/5=1.6$ ,  $y=(9-3 \times 1.6)=4.2$ . Check:  $1.6+8.4=10 \checkmark$

$3x+y=9$  &  $9x+8y=72$ : from 1st  $y=9-3x$ ; sub:  $9x+8(9-3x)=72 \rightarrow 9x+72-24x=72 \rightarrow -15x=0 \rightarrow x=0, y=9$

Check  $x=0, y=9$ :  $x+2y=18 > 10$  ✗ — outside feasible region

$9x+8y=72$  &  $x+2y=10$ : from 2nd  $x=10-2y$ ; sub:  $9(10-2y)+8y=72 \rightarrow 90-18y+8y=72 \rightarrow -10y=-18 \rightarrow y=1.8$ ,  $x=6.4$

Check  $3(6.4)+1.8=21 > 9$  ✗ — outside

Feasible corner points: (0,0), (3,0), (1.6,4.2), (0,5)

Evaluate  $f = 2x+3y$ :

(0,0): 0  $\rightarrow$  Minimum

(3,0): 6

(1.6,4.2):  $3.2+12.6=15.8$

(0,5): 15  $\rightarrow$  need to verify (0,5) feasible:  $3(0)+5=5 \leq 9 \checkmark$ ,  $9(0)+40=40 \leq 72 \checkmark \checkmark$

**ANS: Maximum  $f = 15.8$  at (1.6, 4.2); Minimum  $f = 0$  at (0, 0)**

## ★ ALL THE BEST FOR YOUR EXAMINATIONS ★

This predicted paper was generated by AI analysis of 9 FBISE HSSC-I Mathematics past/model papers (2011–2025). Focus your preparation on Tier-1 topics: Matrix Inverse, Gauss-Jordan, GP/AP problems, Binomial Series proofs, Trig Identities, and General Solution of Trig Equations.

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Chapters with 90%+ probability: Ch-3 (Matrices) · Ch-6 (Sequences) · Ch-10 (Trig Identities) · Ch-12 (App. of Trig) · Ch-14 (Trig Equations)