MATHEMATICS HSSC–I
SECTION – A (Marks 20)
Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

Q.1   Fill the relevant bubble for each part. All parts carry one mark.

1.   In complex numbers, what is the multiplicative inverse of $2i$?

   A. $\frac{-i}{2}$ ○   B. $\frac{1}{2}$ ○

   C. $\frac{-1}{2}$ ○   D. $\frac{i}{2}$ ○

2.   Which one of the following represents the negation of the statement $\sim p \rightarrow q$?

   A. $\sim q \rightarrow p$ ○   B. $\sim q \rightarrow \sim p$ ○

   C. $q \rightarrow \sim p$ ○   D. $p \rightarrow \sim q$ ○

3.   What is the value of α, if $\begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix} = \alpha \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$?

   A.   3   ○   B.   6

   C.   9   ○   D   15   ○

4.   What is the solution set of an equation $x^{-1} + \frac{1}{x^{-1}} = 2$?

   A. $\{1\}$ ○   B. $\{-1, 1\}$ ○

   C. $\{0, -1\}$ ○   D. $\{1, 1\}$ ○

5.   If -1 is a root of $x^3 + kx^2 - x + 2 = 0$, then the value of k is:

   A. -2 ○   B.   2   ○

   C. -4 ○   D.   0   ○

6.   The partial fractions of $\frac{7x+25}{(x+3)(x+4)}$ are:

   A. $\frac{3}{x+4} + \frac{3}{x+3}$ ○   B. $\frac{3}{x+4} + \frac{4}{x+3}$ ○

   C. $\frac{4}{x+4} - \frac{3}{x+3}$ ○   D. $\frac{-4}{x+4} + \frac{3}{x+3}$ ○

7.   For an A.P: $2 + \frac{7}{2} + 5 + \frac{13}{2} + \cdots + a_{10}$, the value of S_{10} is:

   A. $\frac{265}{2}$ ○   B. $\frac{175}{2}$ ○

   C. $-\frac{95}{2}$ ○   D.   195   ○

8. What should be the geometric mean between $\sqrt{2}$ and $3\sqrt{2}$?

A. $\frac{6}{\sqrt{2}}$ ◯   B. $6\sqrt{2}$ ◯

C. $2\sqrt{2}$ ◯   D. $\sqrt{6}$ ◯

9. For what value of n, $^nP_2 = 12$?

A. 4   ◯   B. 3   ◯

C. 12   ◯   D. 6   ◯

10. On tossing a pair of fair coins, what is the probability of appearing both tails?

A. $\frac{3}{4}$ ◯   B. $\frac{2}{4}$ ◯

C. $\frac{1}{4}$ ◯   D. $\frac{2}{3}$ ◯

11. Which one of the following is an expansion of $(1 + x)^{-1}$?

A. $1 - x + x^2 - x^3 + \ldots$ ◯

B. $1 + x - x^2 + x^3 + \ldots$ ◯

C. $1 + x + x^2 + x^3 + \ldots$ ◯

D. $1 - x - x^2 - x^3 + \ldots$ ◯

12. How many terms are there in the expansion of $\left[\left(x - \frac{1}{x}\right)^{\frac{1}{2}}\right]^{60}$?

A. 30   ◯   B. 31   ◯

C. 15   ◯   D. 16   ◯

13. Which one of the following is the simplified form of $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$?

A. $\sec\theta$ ◯   B. $\sec^2\theta$ ◯

C. $2\sec^2\theta$ ◯   D. $2\sec\theta$ ◯

14. What will be the result if simplify $\cos x - \left[\cos\left(x + \frac{\pi}{2}\right) + \cos\left(x - \frac{\pi}{2}\right)\right]$?

A. $\cos x$ ◯   B. $2\cos x$ ◯

C. 0 ◯   D. $(1 - \sqrt{3})\cos x$ ◯

15. If $\theta = \frac{\pi}{2}$, then which one of the following options is true?

A. $\cos 2\theta = \cos^2\theta + \sin^2\theta$ ◯

B. $\sin 2\theta = 2\sin\theta\cos\theta$ ◯

C. $\sin\theta = -2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$ ◯

D. $\cos\theta = \cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right)$ ◯

16. What is the period of a trigonometric function $\sin\left(\frac{\pi x}{2}\right)$?

A. 2   ◯   B. 4   ◯

C. $\frac{1}{2}$ ◯   D. $\frac{1}{4}$ ◯

17. In a triangle ABC, what will be e-radius opposite to vertex A?

A. $\frac{\Delta}{s-a}$ ◯   B. $\frac{\Delta}{s-b}$ ◯

C. $\frac{\Delta}{s-c}$ ◯   D. $\frac{s-a}{\Delta}$ ◯

18. The value of $\sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$ is:

A. $\frac{2}{\sqrt{3}}$ ◯   B. $\frac{-2}{\sqrt{3}}$ ◯

C. $\frac{1}{2}$ ◯   D. $-\frac{1}{2}$ ◯

19. If $\sin^{-1} x + (\sin^{-1} x + \cos^{-1} x) = \pi$, then value of x is:

A. $\frac{1}{2}$ ◯   B. $\frac{1}{\sqrt{2}}$ ◯

C. 0 ◯   D. -1 ◯

20. Which one of the following is a solution set of $\sin x = \frac{1}{2}$, where $x \in [0, 2\pi]$?

A. $\left\{\frac{\pi}{6}, \frac{\pi}{2}\right\}$ ◯   B. $\left\{\frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ ◯

C. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$ ◯   D. $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ ◯

Federal Board HSSC-I Examination
Mathematics Model Question Paper
(Curriculum 2000)

Time allowed: 2.35 hours   Total Marks: 80

Note: Sections 'B' and 'C' comprise pages 1-2 and questions therein are to be answered on the separately provided Answer Book. Write your answers neatly and legibly.

SECTION – B (Marks 48)

Q.2   Attempt any TWELVE parts. All parts carry equal marks. $(12 \times 4 = 48)$

i.   If $z_1 = 1 - 2i, \ z_2 = 2 + 3i, \ z_3 = 4 - 3i$, then find

(a)$\dfrac{\overline{z_2}}{z_3}$ (b)$\overline{z_1} \cdot \overline{z_3}$

and write in the form $a + ib$.

ii.   Convert the following to logical form and prove it by constructing truth table:
$A \cap B = B \cap A$

iii.   If $\begin{vmatrix} x & 1 & x+1 \\ 2 & x & 3 \\ x+1 & 4 & x \end{vmatrix} = 11 - 2x^2$, then find the value of x.

iv.   Find the real roots of $\dfrac{18}{x^4} + \dfrac{1}{x^2} = 4$.

v.   Resolve $\dfrac{3x^2+7x+28}{x(x^2+x+7)}$ into Partial Fractions.

vi.   Prove that cosine is a periodic function and its period is 2π.

vii.   The sum of first thirty terms of an A.P is equal to square of sum of first six terms of the same A.P. Show that $10a + 145d = 12a^2 + 60ad + 75d^2$.

viii.   A committee of 4 persons has to be chosen from 8 boys and 6 girls, consisting of at least one girl. Find the probability that the committee consists of more girls than boys.

ix.   How many automobile license plates can be made, if each plate contains three different letters (from A to Z) followed by four different digits (from 0 to 9)?

x.   Find the first four terms, in ascending powers of x, in the binomial expression of $\dfrac{1}{\sqrt{9+x}}$.

xi.   A regular Pentagon is inscribed in a circle of radius $15 \ cm$. Approximate the perimeter of the Pentagon.

xii.   Prove the trigonometric identity$\dfrac{sin3\theta+sin5\theta+sin7\theta}{cos3\theta+cos5\theta+cos7\theta} = sin5\theta$

xiii.   Draw the graph of $y = \sec x$, $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

xiv.   Prove that $s^2 = \Delta \cot\frac{\alpha}{2} \cot\frac{\beta}{2} \cot\frac{\gamma}{2}$

xv.   Prove that $cot^{-1}\dfrac{119}{120} = 2sin^{-1}\dfrac{5}{13}$.

xvi.   Show that $2\tan^2\theta \ \cos\theta = 3$ can be written in the form $2\cos^2\theta + 3cos\theta - 2 = 0$ and solve the equation $2\tan^2\theta \ \cos\theta = 3$ for $0 \le \theta \le 2\pi$.

SECTION – C (Marks 32)

Note: Attempt any **FOUR** questions. All questions carry equal marks. $(4 \times 8 = 32)$

Q.3 Solve the system of equations by reducing its augmented matrix to the reduced echelon form: $x + 2y + 3z = 3$; $2x + 3y + z = 1$; $3x + y + 2z = 2$

Q.4 In any triangle ABC, with usual notations, prove that:

    (a) $a^2 = b^2 + c^2 - 2bc \, \cos \alpha$

    (b) $\dfrac{a}{\sin \alpha} = \dfrac{b}{\sin \beta} = \dfrac{c}{\sin r}$

Q.5 (a)   Identify the series if its nth term is $a_n = \dfrac{4}{5}\left(\dfrac{2}{3}\right)^{n-1}$

    (b)   What is the sum of first ten terms of the series.

Q.6 If $y = -\dfrac{1}{3} + \dfrac{1}{3^3} + \dfrac{1.3}{2!} \cdot \dfrac{1}{3^5} + \dfrac{1 \cdot 3 \cdot 5}{3!} \cdot \dfrac{1}{3^7} + \ldots \ldots$, then prove that $63y^2 + 84y + 19 = 0$

Q.7 Prove the following trigonometric identity without using calculator/Trigonometric tables.

$$\cos\frac{\pi}{18} \cdot \cos\frac{\pi}{6} \cdot \cos\frac{5\pi}{18} \cdot \cos\frac{7\pi}{18} = \frac{3}{16}$$

Q.8 ABC is a right angled triangle with $m\angle B = 90°$, $m\angle A = 30°$ and $m\overline{AB} = 3cm$

    (a)   Solve the triangle ABC.

    (b)   Find the area $(\triangle)$of triangle ABC.

    (c)   Find the radius (R) of circum-circle of triangle ABC.

    (d)   Find the radius (r) of in-circle of triangle ABC.

MATHEMATICS HSSC-I
Student Learning Outcomes Alignment Chart
(National Curriculum 2000)

Sec-A	Q 1	Contents and Scope	Student Learning Outcomes
	1	Concept of complex numbers and basic operations on them	To know the additive and multiplicative identities of complex numbers and to find their additive and multiplicative Inverses.
	2	Logical proofs of the operation on sets	Introduction to the logical statements and their composition (common connectives, negation, conjunction, disjunction, conditional and bi-conditional).
	3	Determinants and their application in the study of the algebra of matrices	Concept of a determinant of a square matrix, expansion of the determinant up to order 4 (simple cases).
	4	Revision of the work done in previous classes	Solving a quadratic equation in one variable by: (a) factorization (b) completing the square and (c) quadratic formula
	5	Application of Remainder Theorem in the solution of equations	To apply the Remainder Theorem in finding one or two rational roots of cubic and quadratic equations, to use synthetic division in finding depressed equations for solving them.
	6	Partial Fractions	To reduce a fraction into partial fractions when its denominator consists of (a) Linear Factors
	7	Arithmetic Series	To establish the formula for finding the sum up to n-terms of an arithmetic series and be able to apply this formula.
	8	Geometric Mean	To find the Geometric Mean and insert n-G.Ms between two positive real numbers and be able to solve problems based on them.
	9	Permutations	To understand the meaning of permutation of n different things taken r at a time and know the notation ^nP_r or P (n, r).
	10	Probability (Basic concepts and estimation of probability)	To know the formula for finding the probability.
	11	Binomial Theorem for negative integral and rational indices	To apply the theorem in the expansion of the binomial expressions with rational indices as infinite series and arithmetical computations.
	12	Binomial Theorem for positive integral index	To state and prove the binomial theorem for positive integral index, find the number of terms

			and general terms in the expansion of $(a + b)^n$ and apply it to expand positive integral powers of the binomials and find their particular terms (without expansion).
	13	Trigonometric Functions	To establish the following relations between the trigonometric ratios; $cosec\ \theta = 1/\sin\theta$, $sec\ \theta = 1/\cos\theta$, $cot\ \theta = 1/\tan\theta$, $tan\ \theta = \sin\theta / \cos\theta$ $cot\ \theta = \cos\theta/\sin\theta$, $sin2\ \theta + cos2\ \theta = 1$, $1 + tan2\ \theta = sec2\ \theta$ and $1 + cot2\ \theta = cosec2\ \theta$: To be able to apply the above mentioned relations in b)proving the trigonometric identities.
	14	Fundamental formulas of sum and difference of two angles and their applications	b) To establish the formula $\cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta$ and deduction there from for finding the sum and difference of the trigonometric ratios.
	15	Trigonometric ratios of double angles and half angles	To find the values the trigonometric ratios of double and half the angles and apply them.
	16	Periods of trigonometric functions	To know the domains and ranges of the trigonometric functions to have the concept of period of a trigonometric function and the period of the basic trigonometric functions.
	17	Radii of circles connected with triangles	To find the radii of a) Circum circle b) In circle c) Escribed circle of triangles and to solve problems involving these radii.
	18	Inverse trigonometric functions	To know the definition of inverse trigonometric functions their domains and ranges; to know the general and principle trigonometric functions their inverses and their values.
	19	Inverse trigonometric functions	To know the definition of inverse trigonometric functions their domains and ranges; to know the general and principle trigonometric functions their inverses and their values; development of formulas for inverse trigonometric functions and their application.
	20	Solution of trigonometric	To solve trigonometric equations and check their

		equations	answers by substitution in the given equations so as to discard extraneous roots and to make use of the period of trigonometric functions for finding the general solution of the equations.
Sec-B	**Q 2** i	Concept of complex numbers and basic operations on them. Conjugate and its properties	To know the four binary operations on complex numbers and their properties. To know the conjugate and modulus of a complex number $z = x + iy$.
	ii	Logical proofs of the operation on sets	To give formal proofs of the commutative, associative and distributive properties of union and intersection.
	iii	Determinants and their application in the study of algebra of matrices	Expansion of the determinants up to order 4. The use of properties of determinants.
	iv	Solution of equations reducible to quadratic equations in one variable	To solve equation reducible to quadratic equations in one variable.
	v	Partial Fractions	To reduce a fraction in to partial fractions when its denominator consists of (c) non- repeated quadratic factors.
	vi	Periods of Trigonometric functions	To know the domain and ranges of the trigonometric functions to have the concept of period of a trigonometric function and the periods of the basic trigonometric functions.
	vii	Arithmetic Sequence	To find the nth term of an arithmetic progression (A.P) and solve problems pertaining to the terms of an A.P.
	viii	Combinations	To apply combination in solving problems.
	ix	Permutations	To establish the formula for ^nP_r and apply it in solving problems of finding the number of arrangements of n things taken r at a time(when all the n things are different and when some of them are alike) and the arrangements of different things around a circle.
	x	Binomial theorem for negative integral and rational indices	To apply the theorem in the expansion of the binomial expressions with rational indices as infinite series and arithmetical computations.
	xi	Relation between the length of an arc of a circle and the circular measure of its central angle	To establish the rule $\theta = \frac{l}{r}$, where r is the radius of the circle, l is the length of the arc and θ is the circular measure of the central angle of the arc.

	xii	Sum, difference and product of trigonometric ratios	To find the formulas for the following: $\sin\alpha \pm \sin\beta$; $\cos\alpha \pm \cos\beta$; $2\sin\alpha\cos\beta$; $2\cos\alpha\sin\beta$; $2\cos\alpha\cos\beta$ and $-2\sin\alpha\sin\beta$ and to be able to apply.
	xiii	Graphs of trigonometric functions	To draw the graphs of basic trigonometric functions in the domains ranging from -2π to 2π and know that the graphs of these trigonometric functions are repeated depending upon the period of the functions.
	xiv	Radii of circles connected with triangles	To find the radii of a) Circum circle b) In circle c) Escribed circle of triangles and to solve problems involving these radii.
	xv	Inverse trigonometric functions	To know the definition of inverse trigonometric functions their domains and ranges; to know the general and principle trigonometric functions their inverses and their values; development of formulas for inverse trigonometric functions and their applicants; to draw the graphs of inverse trigonometric functions.
	xvi	Solution of trigonometric functions	To solve trigonometric equations and check their answers by substitution in the given equations so as to discard extraneous roots and to make use of the period of trigonometric functions for finding the general solution of the equations.
Sec-C	Q.3	Solvingsimultaneousline ar system of equations	To be able to solve a systems of linear non-homogeneous equations by the use of (b) echelon and reduced echelon form.
	4	Cosine formula Sine formula	► To establish the cosine formula and apply it in the solution of oblique triangles. ► To establish the sine formula to apply it in the solution of oblique triangles
	5	Sequence and Series ►Introduction	To have the concept of a sequence/progression, its term and its domain, different types of sequences with examples to distinguish between arithmetic, geometric and harmonic sequences; and to determine the sequence when its nth term is known.
	6	Binomial Series	To be able to identify the given series as binomial expansion and hence find the sum of the series.
	7	Sum, difference and product of the trigonometric ratios	To find the formulas for the following: $\sin\alpha \pm \sin\beta$; $\cos\alpha \pm \cos\beta$; $2\sin\alpha\cos\beta$; $2\cos\alpha\sin\beta$;

			$2cos\alpha cos\beta$ and $-2sin\alpha sin\beta$ and to be able to apply.
	8	Heights and distances	To be able to use the solution of right triangles in solving the problems of heights and distances. To find the radii of a) Circum circle b) In circle of triangles and to solve problems involving these radii.

MATHEMATICS HSSC-I
Table of Specifications

Topics	1. Number Systems	2. Sets, Functions and Groups	3. Matrices and Determinants	4. Quadratic Equations	5. Partial Fractions	6. Sequences and Series	7. Permutation, Combination and Probability	8. Mathematical Inductions and Binomial Theorem	9. Fundamentals of Trigonometry	10. Trigonometric Identities	11. Fundamentals of Trigonometry	12. Application of Trigonometry	13. Inverse Trigonometric Functions	14. Solution of Trigonometric Equations	Total marks for each assessment objective	% age
Knowledge based	1(1)(1) 2(i)(4)	1(2)(1) 2(ii)(4)			1(6)(1)	5(8)	1(9)(1) 1(10)(1)	1(11)(1)			2(vi)(4)	1(17)(1) 4(8)	1(18)(1)		36	27.2%
Understanding based			1(3)(1) 2(iii)(4) 3(8)	1(4)(1) 2(iv)(4)	2(v)(4)	1(8)(1) 2(vii)(4)		1(12)(1) 6(8)	2(xi)(4)	1(14)(1) 1(15)(1) 2(xii)(4)	1(16)(1)	2(xiv)(4) 8(8)	1(19)(1) 2(xv)(4)	1(20)(1) 2(xvi)(4)	69	52.3%
Application based				1(5)(1)		1(7)(1)	2(viii)(4) 2(ix)(4)	2(x)(4)	1(13)(1)	7(8)	2(xiii)(4)				27	20.5%
Total marks for each topic	05	05	13	06	05	14	10	14	05	14	09	21	06	05	132	100%

KEY:

1(1)(1)

Question No. (Part No.) (Allocated Marks)